FUNCTIONS

<u>Definition</u>

- A **function** is a rule that assigns to each element in a set A exactly one element, called f(x), in a set B

- The set A is called the **domain** of f

- The number f(x) is the **value of f at x**

- The **range** of f is the set of all possible values of f(x) in B as x varies throughout the domain.

- A symbol, such as "x", that represents an arbitrary number in the domain of a function is called an **independent variable**.

- A symbol that represents a number in the range of f is called a **dependent variable**.

<u>Vertical line test</u>

- A curve in the xy-plane is the graph of a function of x if and only if every vertical line intersects the curve **exactly once.**

PROPERTIES OF FUNCTIONS FROM $\mathbb{R}$ TO $\mathbb{R}$

<u>Increasing and Decreasing Functions</u>

- A function f is called **increasing** on an interval I if

- Whenever $x_1 < x_2$ for two numbers x_1 and x_2 in I, then $f(x_1) < f(x_2)$

- A function f is called **decreasing** on an interval I if

- Whenever $x_1 < x_2$ for two numbers x_1 and x_2 in I, then $f(x_1) > f(x_2)$

<u>Symmetric Functions</u>

- If a function f satisfies the property $f(-x) = f(x)$ for every number x in its domain, then f is called an **even** function.

- If a function f satisfies the property $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd** function.

ALGEBRAIC FUNCTIONS

<u>Polynomial Functions</u>

- A function P from $\mathbb{R}$ to $\mathbb{R}$ is called a **polynomial** if

  $P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x^1 + a_0$

  Where n is a nonnegative integer, and the number $a_0, a_1, \ldots, a_n$ are constants called the **coefficients** of the polynomial.

- If the leading coefficient $a_n \neq 0$, then the **degree** of the polynomial is n.

- A polynomial function of degree 0 is called a **constant** function.

- A polynomial function of degree 1 is called a **linear** function.

- A polynomial function of degree 2 is called a **quadratic** function.

- A polynomial function of degree 3 is called a **cubic** function.

<u>Power functions</u>

- A **power** function f is a function of the form $f(x) = x^a$

- If $a = 1/n$ where n is a positive integer, $f(x) = x^{1/n} = \sqrt[n]{x}$ is a **root** function

- If $a = -1$, $f(x) = x^{-1} = 1/x$ is the **reciprocal** function.

<u>Algebraic Functions</u>

- A **rational** function f is a function of the form $f(x) = \dfrac{P(x)}{Q(x)}$

  Where P and Q are two polynomial functions

  The domain of f consists of all values of x where $Q(x) \neq 0$

- An **algebraic** function f is a function that can be constructed using algebraic operations (such as addition, subtraction, multiplication, and taking roots) starting with polynomials.

<u>Piecewise Defined Functions</u>

- A **piecewise defined** function f is a function which is defined with different formulas for different intervals of the domain.