TRANSCENDENTAL FUNCTIONS

- Functions which are not algebraic are called **transcendental** functions. These include, but are not limited to, trigonometric, exponential, and logarithmic function.

EXPONENTIAL FUNCTIONS

- An **exponential** function is a function of the form $f(x) = a^x$ where a is a positive constant.
  if x is a rational p/q, $f(p/q) = a^{p/q} = a^{p.1/q} = (a^p)^{1/q} = \sqrt[q]{a^p}$
- **Laws of exponents**: If a and b are positive numbers and x and y are any real numbers, then
  $a^{x+y} = a^x a^y$
  $a^{x-y} = a^x / a^y$
  $(a^x)^y = a^{xy}$
  $(ab)^x = a^x b^x$
- The number **e** is the irrational number such that the tangent to the graph of $f(x)=e^x$ at (0,1) has a slope of 1.

LOGARITHMIC FUNCTIONS

- if $a>0$ and $a\neq 1$, the exponential function $f(x) = a^x$ is either increasing or decreasing, and so is 1-1 by the horizontal line test. It has therefore an inverse function f^{-1}, called the **logarithmic function with base a**, denoted by $\log_a$. i.e.

  $\log_a x = y \Leftrightarrow a^y = x$
- i.e. for every x in $\mathbb{R}$, $\log_a(a^x) = x$ and $a^{\log_a x} = x$
- in particular, for x=0,1: $0=\log_a(a^0)=\log_a 1$; $1=\log_a(a^1)= \log_a a$
- **Laws of logarithms**: if x and y are positive numbers, then

1. $\log_a(x.y)= \log_a x + \log_a y$
2. $\log_a (x/y) = \log_a x - \log_a y$
3. $\log_a (x^r) = r \log_a x$ (where r is any real number)
4. $\log_a x = (\log_b x) / (\log_b a)$ whenever $a\neq 1$

- **Natural Log**: e is a special base: $\log_e x = \ln x$

TRIGONOMETRIC FUNCTIONS

Definitions

- A **trigonometric** function is a function where the domain or range includes angles.
- The **standard position** of an angle is the one where the vertex is at the origin of the coordinate system, and the initial side on the positive x-axis.
- A **positive** angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side.
- **Negative** angles are obtained similarly with clockwise rotations.

Measuring angles:

- 2π rad = 360^{o}
- so 1 rad = $(180/\pi)^{o} \approx 57.3^{o}$ and $1^{o} = (\pi/180)$ rad ≈ 0.017 rad

Calculating length of arcs:

To find length a of arc of angle θ rad, solve for a:
i.e. $a = r\theta$

Sin and cos

- **Definitions of cos and sin**
  Given a circle C of center (0,0) and radius 1 and an angle θ
  Let P be the point where the terminal side of the angle θ in standard position intersects with C.
  cos θ = x coordinate of P.
  sin θ = y coordinate of P.

  Since C is the graph of all points (x,y) with the property $x^2+y^2=1$
  then $\cos^2\theta + \sin^2\theta = 1$

- **Properties**
  $\cos(-\theta) = \cos\theta$ i.e. cos is even
  $\sin(-\theta) = -\sin\theta$ i.e. sin is odd
  $\cos(\theta+2\pi) = \cos\theta$
  $\sin(\theta+2\pi) = \sin\theta$

- **General Definition:**
  Given an angle θ and a point P(x,y) on the terminal side of the angle.
  Let r be the distance between P and the vertex of the angle.
  Then cos θ = x/r and sin θ = y
- **Alternate definition for acute angles**
  For an acute angle θ
  cos θ = adj/hyp and sin θ = opp/hyp

Tan

- Given an angle θ, define tan θ = (sin θ) / (cos θ)

Inverse Functions:

- arcsin x = $\sin^{-1}$ x for x in domain [-π/2,π/2]
- arcos x = $\cos^{-1}$(x) in domain [0, π]
- arctan x = $\tan^{-1}$ x for x in domain [-π/2,π/2]

Other Functions:

Given an angle θ, define:

- sec θ = 1 / (cos θ)
- csc θ = 1 / (sin θ)
- cot θ = 1 / (tan θ) = (cos θ) / (sin θ)