

## Definitions

- A function  $f$  has an **absolute maximum** (or **global maximum**) at  $c$  iff  $\forall x \in D, f(c) \geq f(x)$ , where  $D$  is the domain of  $f$ .  
The number  $f(c)$  is called the **maximum value** of  $f$  on  $D$ .
- A function  $f$  has an **absolute minimum** (or **global minimum**) at  $c$  iff  $\forall x \in D, f(c) \leq f(x)$ , where  $D$  is the domain of  $f$ .  
The number  $f(c)$  is called the **minimum value** of  $f$  on  $D$ .
- The maximum and minimum values of  $f$  are called the **extreme values** of  $f$ .
- A function  $f$  has an **local maximum** (or **relative maximum**) at  $c$  iff  $\forall x \in (a,b), f(c) \geq f(x)$ , where  $(a,b)$  is an interval containing  $c$ .
- A function  $f$  has an **local minimum** (or **relative minimum**) at  $c$  iff  $\forall x \in (a,b), f(c) \leq f(x)$ , where  $(a,b)$  is an interval containing  $c$ .

## Extreme Value Theorem

- If  $f$  is continuous on a closed interval  $[a,b]$ , then  $f$  attains an absolute maximum value  $f(c)$  and an absolute minimum value  $f(d)$  at some numbers  $c$  and  $d$  in  $[a,b]$ .

## Critical Numbers

- **Fermat's Theorem:** If  $f$  has a local maximum or minimum at  $c$ , and if  $f'(c)$  exists, then  $f'(c) = 0$ .
- A **critical number** of a function  $f$  is a number  $c$  in the domain of  $f$  such that either  $f'(c) = 0$  or  $f'(c)$  doesn't exist.
- If  $f$  has a local maximum or minimum at  $c$ , then  $c$  is a critical number of  $f$ .

## Closed Interval Method

- To find the absolute maximum and minimum values of a continuous function  $f$  on a closed interval  $[a,b]$ :
  - Calculate  $f(a)$  and  $f(b)$
  - Calculate the values of  $f(x)$  for all critical points  $x$  in  $[a,b]$
  - The largest of these values is the absolute maximum value in  $[a,b]$  and the smallest is the absolute minimum value in  $[a,b]$ .