

Rolle's Theorem

Let f be a function that satisfies the following hypotheses:

- f is continuous on the closed interval $[a,b]$
- f is differentiable on the open interval (a,b)
- $f(a) = f(b)$

Then there is a number c in (a,b) such that $f'(c) = 0$

The Mean Value Theorem

Let f be a function that satisfies the following hypotheses:

- f is continuous on the closed interval $[a,b]$
- f is differentiable on the open interval (a,b)

Then there is a number c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

or equivalently $f(b) - f(a) = f'(c)(b - a)$

Constant Functions

- if $f'(x) = 0$ for all x in an interval (a,b) , then f is constant on (a,b)
- if $f'(x) = g'(x)$ for all x in an interval (a,b) , then $f - g$ is constant on (a,b)

i.e. $(\forall x \in (a,b), f'(x) = g'(x)) \Rightarrow \exists c \in \mathbb{R}, \forall x \in (a,b), f(x) = g(x) + c$