First Derivatives and Graphs

Increasing/Decreasing Test:

- $(\forall x \in (a,b), f'(x)=0) \Rightarrow$ f is constant on (a,b)
- $(\forall x \in (a,b), f'(x)>0) \Rightarrow$ f is increasing on (a,b)
- $(\forall x \in (a,b), f'(x)<0) \Rightarrow$ f is decreasing on (a,b)

First Derivative Tests:

Suppose that c is a critical number of a continuous function f.

- If f′ changes from positive to negative at c, then f has a local maximum at c.
- If $(\forall x<c\ f'(x)>0) \wedge (\forall x>c\ f'(x)<0)$, then f has an absolute maximum at c.
- If f′ changes from negative to positive at c, then f has a local minimum at c.
- If $(\forall x<c\ f'(x)<0) \wedge (\forall x>c\ f'(x)>0)$, then f has an absolute minimum at c.
- If f′ doesn't change sign at c, then f has no local maximum or minimum at c.

Second Derivatives and Graphs

Concavity

- If the graph of a function f lies above all its tangents on an interval I, then it is called **concave upward (CU)** on I.
- If the graph of a function f lies below all its tangents on an interval I, then it is called **concave downward (CD)** on I.
- A point P on the graph of a function f is called an **inflexion point** if f is continuous there and the concavity of the curve changes at c.

Concavity Test

- If $\forall x \in I$, $f''(x)>0$, then the graph of f is concave upward on I.
- If $\forall x \in I$, $f''(x)<0$, then the graph of f is concave downward on I.
- If a point c in an interval I is s.t $f''(x)$ has one sign for all x smaller than c and the opposite sign for all x larger than c, then c is an inflexion point of f.

Second Derivative Test:

Suppose f″ is continuous near c.

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c
- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c