

### Indeterminate Forms

- Given two functions  $f$  and  $g$ .
- Let  $a$  be a point where the limits of these functions at  $a$  are  $0$  or  $\pm\infty$
- The limits of algebraic combinations of these functions at  $a$  may or may not exist and are called indeterminate forms.

### Indeterminate Quotients

- If  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  is of type  $\frac{0}{0}$
- If  $\lim_{x \rightarrow a} f(x) = \pm\infty$  and  $\lim_{x \rightarrow a} g(x) = \pm\infty$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  is of type  $\frac{\infty}{\infty}$

### Indeterminate Products:

- If  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = \pm\infty$ , then  $\lim_{x \rightarrow a} [f(x) \cdot g(x)]$  is of type  $0 \cdot \infty$

### Indeterminate Differences:

- If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = \pm\infty$ , then  $\lim_{x \rightarrow a} [f(x) - g(x)]$  is of type  $\infty - \infty$

### Indeterminate Powers:

- If  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} [f(x)]^{g(x)}$  is of type  $0^0$
- If  $\lim_{x \rightarrow a} f(x) = \pm\infty$  and  $\lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} [f(x)]^{g(x)}$  is of type  $\infty^0$
- If  $\lim_{x \rightarrow a} f(x) = 1$  and  $\lim_{x \rightarrow a} g(x) = \pm\infty$ , then  $\lim_{x \rightarrow a} [f(x)]^{g(x)}$  is of type  $1^\infty$

### L'Hospital's Rule

Let  $f$  and  $g$  be differentiable functions, and  $g'(x) \neq 0$  near  $a$  (except possibly at  $a$ )

Suppose that:  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  is of indeterminate form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$

Then if  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  exists or is  $\pm\infty$ ,  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$