

1. Find domain  $D = \{\text{all values of } \mathbb{R} \text{ where } f \text{ is defined}\} = \{x \in \mathbb{R} \mid \exists! y \in \mathbb{R}, y=f(x)\}$

2. Limits where f is undefined or not continuous

Determine what happens to f as it approaches the limits of its domain or its continuity.

- Calculate  $\lim_{x \rightarrow a^\pm} f(x)$  whenever f(a) is not defined or f not continuous at a
  - If this limit is  $\pm\infty$ , there will be a vertical asymptote
- Calculate  $\lim_{x \rightarrow \pm\infty} f(x)$ 
  - If this limit is a constant c, there will be a horizontal asymptote.
  - If  $\lim_{x \rightarrow \infty} [f(x) - (mx + b)] = 0$ , then the line  $y=mx+b$  is a slant asymptote.

3. Symmetry and Repetition

- Curves of even functions:  $\forall x \in D \ f(-x)=f(x)$  are symmetric about the y-axis
- Curves of odd functions:  $\forall x \in D \ f(-x)=-f(x)$  are symmetric about the origin
- Curves of periodic functions:  $\forall x \in D \ f(x+p)=f(x)$  for a constant p are repeated over consecutive intervals of size p.

4. Intervals of Increase and Decrease: compute  $f'(x)$

- f is increasing in intervals where  $f'(x) > 0$
- f is decreasing in intervals where  $f'(x) < 0$

5. Local and Absolute Maxima and Minima

Find all the critical numbers of f:  $f'(x)$  is 0 or undefined. Use first or second derivative tests to determine if there are extreme values at these points

6. Concavity and Points of Inflection: compute  $f''(x)$

- f is concave up in intervals where  $f''(x) > 0$
- f is concave down in intervals where  $f''(x) < 0$
- Inflection points occur where f changes concavity.

7. Intercepts

- f intercepts y-axis at  $(0, f(0))$
- f intercepts x-axis at  $(x, 0)$  s.t.  $f(x)=0$

8. Sketch curve