RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

MTH 210 **Final Exam** **April 18, 2006**

Total marks: 50 **Time allowed: 3 hrs.**

NAME (Print): _____ **STUDENT #:** _____

Circle your Lab Section:

Monday 11 BUS 300 021	Wednesday 10 EPH 112 011	Wednesday 10 BUS 210 031

Instructions:

- Verify that your paper contains **??**Last Question questions on **??**Last Page pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cellphones, pagers and ipods must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the exam.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

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6 Mk 1. Consider the following (recursive) definition of *gurgle graphs*:

(a) K_2 (the simple graph consisting of 2 points and 1 edge) is a gurgle graph.

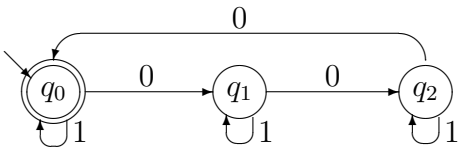
(b) If $G = (V, E)$ is a gurgle graph and $x \in V$ has degree one and $y \notin V$ then $G' = (V \cup \{y\}, E \cup \{x, y\})$ is a gurgle graph.

(c) No other graph is a gurgle graph other than those created by the rules above.

Show by structural induction that every gurgle graph has exactly two vertices of degree one.

2. For the following FSA, M, find the language accepted by M.
(Give your answer as both a regular expression and in English.)

[5 Mk]



3. Let $L = (a \vee b)^* aa$, the language of strings of as and bs which end in aa. Give the state diagram of an FSA M, with fewer than 5 states, such that $L(M) = L$.

[4 Mk]

4. In each of the questions below circle the appropriate answer.
(Warning: marks may be deducted for incorrect answers).

6 Mk

(a) Every Finite language is regular. **T** **F**

(b) Given a finite set Σ, the set Σ^* is countable. **T** **F**

(c) Every language L over an alphabet Σ is recognizable. **T** **F**

(d) Every recognizable language is decidable. **T** **F**

(e) Every regular language is decidable. **T** **F**

(f) The union of any two regular languages is regular. **T** **F**

6 Mk 5. Use the Pumping Lemma to show that $L = \{a^n b^m a^{n+m} \mid m, n \in \mathbb{N}\}$ is not regular.

6. (a) A certain application compares two times by returning the number of seconds between them. Use the mod and div functions to find formulae that determine what minute, what hour and what day of the week it will be x seconds from now, assuming now is 9:27:34 am on a Tuesday.

5 Mk

Use a 24 hour clock (i.e., 0hr = 12 am, 1hr = 1am, . . . , 12hr = 12 noon, 13hr = 1pm,. . . , 23hr = 11pm). Number the days starting with Monday = 0. There are 60 seconds in a minute and 60 minutes in an hour. Do not evaluate any additions or multiplications, leave them as $a + b$ or $a \times b$.

Formula for the minute: _____

Formula for the hour: _____

Formula for the day: _____

(b) Use the Euclidian Algorithm to find gcd(3576, 1536). Show each step of the execution of the algorithm.

4 Mk

2 Mk

7. (a) How many ways are there of rearranging the letters of the word MEETER?

3 Mk

(b) How many of these begin with E or T?

2 Mk

(c) What is the probability that a random word chosen from the letters of MEETER will start with E or with T?

8. Note: For this question leave your answers as fractions, possibly containing powers. However your answers should not contain any factorials or summation signs.

   In a multiple choice paper there are 6 questions, each of which has one of 3 possible answers.

   (a) If someone was to randomly guess the answers, use the binomial distribution to determine the probability that they get exactly 4 correct.

   2 Mk

   (b) If they know the correct answer to two questions, but guess the rest, use the binomial distribution to determine the probability that they get exactly 4 correct.

   2 Mk

   (c) If 1 mark is awarded for each correct answer and 1 mark is deducted for each incorrect answer what is the expected mark if all questions are guessed?
   (Be sure to show all your work.)

   3 Mk