RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

MTH 210 **Midterm Test** **March 1, 2007**

Total marks: 50 **Time allowed: 110 Minutes**

NAME (Print): _____ **STUDENT #:** _____

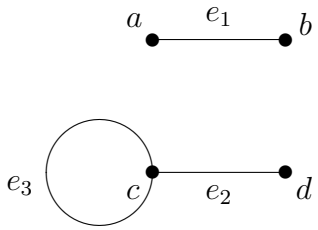
Circle your Lab Section:

011	021	031
ENG 102	ENG 105	LG 06

Instructions:

- Verify that your paper contains 6 questions on 6 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cellphones, pagers and ipods must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.

1.



G

6 Mk

For the graph G given above give the following, or explain why the graph does not have the given property. (Quote any theorems you use.)

(a) On the graph circle the connected components of G.

(b) What are the endpoints of e_2?

(c) Find a loop in G

(d) Give the degree sequence of G.

(e) Find a walk from c to d which is not a path.

(f) Find a Hamiltonian circuit in G.

2. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property. [10 Mk]

   (a) A graph with degree sequence 1,1,2,2.

   (b) A graph with total degree 11.

   (c) The complete bipartite graph $K_{2,3}$.

   (d) A tree with 7 vertices and 7 edges.

   (e) A full binary tree with 5 vertices and 3 leaves.

3. Find all simple graphs on three vertices up to isomorphism. [4 Mk]

4. Consider the sequence

$$b_0 = 1;$$
$$b_i = 3b_{i-1} + 2 \quad i \geq 1.$$

(a) Find b_1, b_2, b_3 and b_4. Keep your intermediate answers as you will need them for the next part of the question.

4 Mk

(b) Use iteration to find an analytic formula for the sequence b_n. Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums. Quote any formulas or rules that you use.

6 Mk

5. Let e_n be the sequence defined by

$$e_0 = 1;$$
$$e_n = \frac{e_{n-1}}{1 + 2e_{n-1}}, \quad n \geq 1.$$

10 Mk

Use weak induction to show that $e_n = \dfrac{1}{2n+1}$ for every $n \geq 0$.

Be sure to lay out your proof clearly and correctly.

6. Use strong mathematical induction to show that postage of n cents can be made up by using some combination of 3 or 7 cent stamps, whenever $n \geq 12$.

10 Mk

In other words:
Show that any integer greater than 11 can be expressed as a sum of multiples of 3 and 7.

Be sure to lay out your proof clearly and correctly.