RYERSON UNIVERSITY

DEPARTMENT
OF
MATHEMATICS

MTH 210	**Midterm Test**	**February 27, 2009**
Total marks: 65		**Time allowed: 110 Minutes**

NAME (Print): ____________________ **STUDENT #:** ____________

Circle your Lab Section:

011	021	031
VIC 200	VIC 205	VIC 608

Instructions:

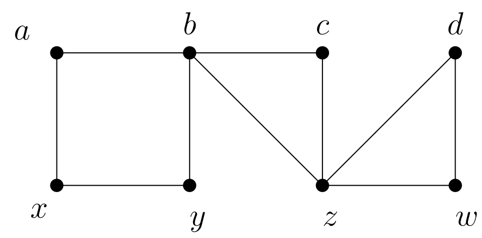
- Verify that your paper contains 7 questions on 7 pages.
- You are allowed an $8\frac{1}{2} \times 11$ formula sheet written on both sides.
- No other aids allowed. Electronic devices such as calculators, cellphones, pagers and ipods must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.
- You may assume the following formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$$

- Recall the definition of divides:

$$a \mid b \Leftrightarrow \exists\, m \in \mathbb{Z} \text{ such that } b = am.$$

1.



G

For the graph G given above give the following, or explain why the graph does not have the given property. (Quote any theorems you use.)

5 Mk (a) Find the order, size, total degree and degree sequence.

Order	Size	Total Degree

Degree Sequence:

1 Mk (b) Is G simple?

1 Mk (c) How many connected components does G have?

1 Mk (d) Find a walk from c to d which is not a path.

2 Mk (e) Find an Eulerian circuit in G.

2 Mk (f) Draw a spanning tree of G.

2. For each of the following either explain why the specified graph cannot exist (quote any theorems you use), or draw a graph with the given property.

2 Mk (a) A simple graph with degree sequence $1, 1, 2, 2, 3$.

2 Mk (b) A simple graph with degree sequence $1, 2, 2, 2, 3$.

2 Mk (c) A bipartite cycle (Recall: a cycle is a simple circuit).

2 Mk (d) The path of length 4, P_5 and its complement $\overline{P_5}$.

2 Mk (e) A $3-$regular graph on 4 points.

2 Mk (f) A full binary tree with 6 vertices and 4 leaves.

2 Mk (g) A binary tree with height 2 and 3 leaves

4 Mk

3. List all simple connected graphs on 4 points, up to isomorphism.

8 Mk

4. In the following f, g and h are functions and n is a variable. Answer true or false in each case by circling the appropriate answer. Note that marks may be deducted for incorrect answers.

(a) $n \in O(\log n)$ **T** **F**

(b) $\dfrac{n^6 - 50n^5 + 200n^3 - 357n^2 + 5489}{5n - 20} \in o(n^6)$ **T** **F**

(c) $50n^5 + 200n^3 - 357n^2 + 5489 \in O(n^6)$ **T** **F**

(d) $2^n \in \theta(3^n)$ **T** **F**

(e) If $f \in O(g)$ then $g \in \Omega(f)$ **T** **F**

(f) If $f \in O(h)$ and $g \in O(h)$ then $f + g \in O(h)$ **T** **F**

(g) If $f \in O(h)$ and $g \in O(h)$ then $fg \in O(h)$ **T** **F**

(h) If $f \in O(h)$ and $g \in O(h)$ then $fg \in \Omega(h)$ **T** **F**

5. Consider the sequence

$$a_1 = 2;$$
$$a_n = a_{n-1} + n + 1, \quad n > 1.$$

4 Mk

(a) Find $a_2,\ a_3,\ a_4$ and a_5. Keep your intermediate answers as you will need them for the next part of the question.

5 Mk

(b) Use iteration to find an analytic formula for the sequence a_n. Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums. Quote any formulas or rules that you use.

9 Mk

6. Show by weak induction that 8 divides $3^{2n} - 1$, for every $n \geq 0$.

   Be sure to lay out your proof clearly and correctly.

9 Mk

7. Let b_n be the sequence defined by $b_0 = 3$, $b_1 = 5$ and

$$b_n = 2b_{n-1} + 3b_{n-2}, \text{ for } n > 1.$$

Use strong induction to show that

$$b_n = 2 \cdot (3)^n + (-1)^n.$$

Be sure to lay out your proof clearly and correctly.